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INFLUENCE OF THE HALL EFFECT ON PLASMA COMPRESSIBILITY IN A STRONG TOROIDAL MAGNETIC FIELD

V. P. Zhukov and G. Fuchs[1]

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The development of the tearing instability is studied in the presence of a high toroidal magnetic field and a high plasma conductivity. The variation of the plasma density is shown to be significant in this case.

We study some aspects of the influence of the Hall effect on reconnection processes in a cylindrical (helical) geometry in the approximation of a strong magnetic field directed along the cylinder's axis. The interest in this problem is associated with problems that arise in the description of sawtooth oscillations in tokamaks. It is known that, giving, in principle, the true pattern of these oscillations, Kadomtsev's simple two-dimensional model [1, 2] conflicts with experimental results [3, 4]: the reconnection time turns out to be smaller than the predicted one [1, 2], and the complete reconnection is not observed in some experiments. Numerical modeling of three-dimensional problems in the approximation of single-fluid magnetic hydrodynamics [5, 6] yields results which are close to those in [1, 2].

Attempts have recently been made to explain the discrepancies between theory [1, 2] and experiment by the influence of the Hall effect [7–12]. The effect of the electron pressure gradient in the Ohm's generalized law was examined in [7, 8]. The expansion of the initial equations in the parameter $R \ll 1$, which is equal to the ratio of the large-to-small radius of a tokamak, was used in [7–12]. The density variation was assumed to be zero or small.

In the present paper, we show that the formal expansion of the MHD equations in the parameter R allows one to derive equations according to which

(1) the density variation is small (of the order of R^{-1});

(2) in contrast to [7, 8], the term containing the electron pressure in the Ohm's generalized law disappears. This is natural because the presence of the pressure and density gradients is necessary for this term to exert an effect on the current;

(3) a complete pressure-containing term appears in the simplified equation for the vector potential, and this term has no concern with ∇p_e in the Ohm's generalized law and magnetic-field freezing into the electron plasma component.

However, estimates and numerical simulation results show that, according to these equations, the density variation is insignificant if the parameter $\alpha R^{-3/2}\nu^{-1/2}$ is small (α is the Hall coefficient equal to the ratio of the ion dispersion dimension c/ω_{pi} to the small tokamak radius, and ν is the coefficient of magnetic viscosity). For most tokamaks, the parameter $\alpha R^{-3/2}\nu^{-1/2}$ is large. Therefore, the density variation can be marked, and the formal expansion turns out to be not applicable because of the existence of thin current layers.

Here we give a system of equations in which expansion in the parameter R is used, but the variation in density is not assumed to be small. Therefore, the expansion in the parameter R has no significant advantages for numerical analysis compared with the solution of the initial equations.

Institute of Computational Technologies, Siberian Division, Russian Academy of Sciences, 630090 Novosibirsk. [1]Institute for Plasma Physics, Juelich, Germany. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 2, pp. 3–9, March–April, 1998. Original article submitted May 24, 1996; revision submitted July 9, 1996.

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Initial Equations. We use the magnetohydrodynamic equations which incorporate the Hall effect as the initial equations [13, 14]. We ignore the electron inertia, which has no importance for the effect in which we are interested. In the common notation, the system is of the form

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}\right)=-\nabla(p_e+p_i)+\mathbf{j}\times\mathbf{H},\quad \frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{V})=0,\quad \mathbf{j}=\nabla\times\mathbf{H},$$

$$\mathbf{E}=-\mathbf{V}_e\times\mathbf{H}+\nu\mathbf{j}-\alpha\rho^{-1}\nabla p_e,\quad \mathbf{V}_e=\mathbf{V}-\alpha\rho^{-1}\mathbf{j},\quad \frac{\partial\mathbf{H}}{\partial t}=-\nabla\times\mathbf{E}.$$

In the cylindrical coordinate system (r,φ,z), in the case of helical symmetry $\partial/\partial z=-R^{-1}\partial/\partial\varphi$ these equations take the form

$$\rho\left(\frac{\partial V_g}{\partial t}+(\mathbf{V}\nabla)V_g\right)=\operatorname{div}(\mathbf{H}H_g);\tag{1}$$

$$\rho\left(\frac{\partial V_r}{\partial t}+(\mathbf{V}\nabla)V_r-\frac{V_\varphi^2}{r}\right)=-\frac{\partial(p_e+p_i)}{\partial r}+\left(1+\frac{r^2}{R^2}\right)^{-1}\left(-\frac{\partial H_g^2/2}{\partial r}+j_g\frac{\partial A_g}{\partial r}\right);\tag{2}$$

$$\rho\left(\frac{\partial V_\varphi}{\partial t}+(\mathbf{V}\nabla)V_\varphi+\frac{V_\varphi V_r}{r}\right)=-\frac{1}{r}\frac{\partial(p_e+p_i)}{\partial\varphi}$$
$$+\left(1+\frac{r^2}{R^2}\right)^{-1}\left(-\frac{1}{r}\frac{\partial H_g^2/2}{\partial\varphi}+j_g\frac{1}{r}\frac{\partial A_g}{\partial\varphi}+\frac{r}{R}\operatorname{div}(\mathbf{H}H_g)\right);\tag{3}$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{V})=0;\tag{4}$$

$$\frac{\partial A_g}{\partial t}+(\mathbf{V}_e\nabla)A_g=-\nu j_g;\tag{5}$$

$$\frac{\partial H_z}{\partial t}+\operatorname{div}(\mathbf{V}_eH_z)=\operatorname{div}(\mathbf{H}V_{ez})+\nu\Delta_sH_z-\alpha\operatorname{div}(\rho^{-1}(\mathbf{e}\times\nabla)p_e),\qquad \mathbf{e}=(0,0,1);\tag{6}$$

$$\mathbf{V}_e=\mathbf{V}-\alpha\rho^{-1}\mathbf{j},\quad H_s=-\frac{\partial A_g}{\partial r},\quad H_r=\frac{1}{r}\frac{\partial A_g}{\partial\varphi},\quad j_s=-\frac{\partial H_g}{\partial r},$$
$$j_r=\frac{1}{r}\frac{\partial H_g}{\partial\varphi},\quad j_g=-\Delta_sA_g+2\frac{H_z}{R}.\tag{7}$$

Here f_s and f_g are the quantities related to the components of the vector $\mathbf{f}$ as follows:

$$f_s=f_\varphi-\frac{r}{R}f_z,\quad f_g=f_z+\frac{r}{R}f_\varphi;\quad \Delta_s=\left(1+\frac{r^2}{R^2}\right)\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right);$$
$$\operatorname{div}(\mathbf{f})=\frac{1}{r}\frac{\partial rf_r}{\partial r}+\frac{1}{r}\frac{\partial f_s}{\partial\varphi};\quad (\mathbf{f}\nabla)g=f_r\frac{\partial g}{\partial r}+f_s\frac{\partial g}{\partial\varphi};$$

A_g is the g component of the vector potential, H_z and H_g are the z and g components of the magnetic field, and $\mathbf{V}_e$ is the electron velocity. System (1)–(7) is given in dimensionless variables [13]. The characteristic transverse plasma dimension a (the small radius of a tokamak) is used as the scale of length, the Alfvén velocity calculated over a toroidal magnetic field: $V_\mathrm{A}=H_z/\sqrt{4\pi\rho}$ > as the scale of velocity, a/V_A as the scale of time, and H_z as the scale of the magnetic field.

Equations (1)–(7) should be supplemented by the equations for an electron p_e and an ion p_i pressure, whose concrete forms will be given below.

Initial and Boundary Conditions. As the initial conditions, we use the following ones:

$$\rho=1,\quad H_z=1,\quad V_z=0,\quad H_r=0,\quad H_s=\frac{r}{R}\left(\frac{1-(1-r^2)^{q+1}}{qr^2}-1\right).\tag{8}$$

Therefore, we have a situation with a neutral layer. We have $H_s>0$ near the coordinate axis and $H_s<0$ for large radii. The position of the neutral surface ($H_s=0$) depends on the quantity q.

The plasma pressure at the initial moment of time is assumed to be such that the plasma equilibrium in a magnetic field is ensured. The equilibrium is broken by a small velocity perturbation whose concrete form plays no part.

The problem was solved in the domain $0 \leqslant r \leqslant 1$, $0 \leqslant \varphi \leqslant 2\pi$. The boundary $r = 1$ is assumed to be a conducting surface:

$$H_r = 0 \ \ (A_g = \text{const}), \quad V_r = 0, \ \ p_{e,i} = \text{const}, \quad E_\varphi = V_{er} H_z - V_{ez} H_r - \nu \frac{\partial H_z}{\partial r} - \frac{1}{\rho r}\frac{\partial p_e}{\partial \varphi} = 0.$$

Approximation of Large R. We simplify the formulated problem assuming that $R^{-1} \ll 1$, similarly to [1, 2], but, in contrast to [1, 2], the density variation is not assumed to be small. Here we have the following scale of the quantities:

$$O(1): \ H_z, \rho, \nabla_\perp, \qquad O(R^{-1}): \ A_g, V_r, V_\varphi, \frac{\partial}{\partial t}, \qquad O(R^{-2}): \ H_z - 1, p_{e,i}, j_r, j_z, V_z.$$

In the first approximation, we obtain the Kadomtsev model:

$$\rho\Big(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V}\Big) = -\nabla(p_e + p_i + H_g) + j_g \nabla A_g,$$
$$\frac{\partial A_g}{\partial t} + (\mathbf{V}\nabla)A_g = -\nu j_g, \quad \operatorname{div}\mathbf{V} = 0, \quad \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0. \tag{9}$$

If $\rho = 1 + O(R^{-1})$ at the initial moment of time, the deviation of ρ from unity will be of the order of $O(R^{-1})$ in the subsequent moments as well. Hence, for the vorticity $\omega = (\mathbf{e} \operatorname{rot} \mathbf{V})$, we write the equation

$$\frac{\partial \omega}{\partial t} + (\mathbf{V}\nabla)\omega = \operatorname{div}(\mathbf{H} j_g). \tag{10}$$

The second approximation makes it possible to take into account the effects of interest. It has the form

$$\rho\Big(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V}\Big) = -\nabla(p_e + p_i + H_g) + j_g \nabla A_g; \tag{11}$$

$$\frac{\partial A_g}{\partial t} + (\mathbf{V}_e\nabla)A_g = -\nu j_g, \quad j_g = \frac{2}{R} - \Delta A_g; \tag{12}$$

$$\operatorname{div}\mathbf{V} = -\alpha \operatorname{div}(\rho^{-1}\{\mathbf{e} \times \nabla(p_e + H_g) + \mathbf{H} j_g\}); \tag{13}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0; \tag{14}$$

$$\mathbf{H} = -\mathbf{e} \times \nabla A_g, \quad \mathbf{j} = -\mathbf{e} \times \nabla H_g, \quad \mathbf{V}_e = \mathbf{V} - \alpha\rho^{-1}\mathbf{j}. \tag{15}$$

Here, by $\mathbf{V}$, $\mathbf{H}$, and $\mathbf{j}$, we mean their transverse (r and s) components, and Δ is the Laplace standard operator. We note that $V_z = 0$ in this approximation. For $\alpha = 0$, the model (11)–(15) becomes the Kadomtsev model.

We shall estimate how the density varies according to Eqs. (11)–(15). It follows from (13) that

$$\mathbf{V} = \mathbf{V}_0 - \alpha\rho^{-1}(\mathbf{e} \times \nabla(p_e + H_g) + \mathbf{H} j_g), \tag{16}$$

where $\mathbf{V}_0$ is the vector whose divergence is zero. The quantity $\mathbf{V}_0$ can be found from a joint solution of Eqs. (11) and (13).

Substituting (16) into (14), we obtain

$$\frac{\partial \rho}{\partial t} + (\mathbf{V}_0\nabla)\rho = \alpha \operatorname{div}(\mathbf{H} j_g). \tag{17}$$

Representing ρ in the form $\rho = 1 + \alpha\rho_*$, for ρ_* we have the same equation as that for vorticity in the Kadomtsev model (10). Thus, if the coefficient α is sufficiently small, the formula $\rho \approx 1 + \alpha\omega$ holds. Since $\omega \sim R^{-1}$, the formal variation in density is small. However, as is known, the current at small ν is accompanied by the formation of a current layer of width of the order of $\nu^{1/2}$. The plasma flows into this layer with a velocity of the order of $\nu^{1/2}$ and flows out along this layer with a velocity approximately equal to the Alfvén one calculated for a poloidal magnetic field [1, 2]. Thus, the vorticity is $\omega \sim \nu^{-1/2}$.

It is easy to see that the Kadomtsev model possesses the following property. If $\mathbf{V}(t, \mathbf{r})$ and $A_g(t, \mathbf{r})$ are the solutions of (9) and (10) for $R = R_*$, $\nu = \nu_*$, and the initial data (8), the functions $R_*\mathbf{V}(R_* t, \mathbf{r})$ and $R_* A_g(R_* t, \mathbf{r})$ will be the solutions of these equations for $R = 1$ and $\nu = \nu_* R_*$. Therefore, we have the scaling $\omega \sim R^{-3/2}\nu^{-1/2}$ for ω. Correspondingly, the density variation will be of the order of $2\alpha R^{-3/2}\nu^{-1/2}$. This quantity is very large for real tokamaks. For example, for the typical parameters of a TEXTOR facility ($\alpha \approx 0.05$ and $R \approx 3.8$, and the Coulomb value $\nu \sim 10^{-8}$) it is equal to 60.

Approximation of Small Density Variation. We shall clarify at which parameters the density variation is significant. To do this, we assume that the value of α is sufficiently small and, therefore, $\rho - 1 \sim R^{-1}$ and $\operatorname{div}\mathbf{V} \sim R^{-2}$. In essence, this will be the formal expansion of Eqs. (1)–(7) with initial parameters (8) in the parameter R.

Equation (13) in this case takes the form

$$\operatorname{div}\mathbf{V} = -\alpha \operatorname{div}(\mathbf{H} j_g). \tag{18}$$

Thus, the effects associated with the electron pressure gradient in the Ohm's law coincide. We recall that (13) is derived from Eq. (6) containing ∇p_e.

With allowance for (18), with an accuracy required with respect to R^{-1}, from (11) we obtain [$\omega = (\mathbf{e}\operatorname{rot}\mathbf{V})$]

$$\begin{gathered}\frac{\partial \omega}{\partial t} + (\mathbf{V}\nabla)\omega = (1 + \alpha\omega)\operatorname{div}\frac{\mathbf{H} j_g}{\rho} - \operatorname{div}\frac{\operatorname{rot}(P\mathbf{e})}{\rho},\\ \mathbf{V} = -(\mathbf{e} \times \nabla\psi_\omega) - \alpha \mathbf{H} j_g, \quad \Delta\psi_\omega = -\omega + \alpha \operatorname{div}(j_q \nabla A_g).\end{gathered} \tag{19}$$

The quantity $P = p_e + p_i + H_g$ can be calculated with sufficient accuracy, using the divergence of (11) and setting $\rho = 1$ and $\operatorname{div} V = 0$:

$$\Delta P = \operatorname{div}(j_g \nabla A_g) - \frac{\partial V_k}{\partial x_l}\frac{\partial V_l}{\partial x_k}. \tag{20}$$

For density, we write

$$\frac{\partial \rho}{\partial t} - (\mathbf{e} \times \nabla\psi_\omega)\nabla\rho = \alpha \operatorname{div}(\mathbf{H} j_g). \tag{21}$$

For the expression $(\mathbf{V}_e\nabla)A_g$ in (12), with allowance for (15) and the fact that $(\mathbf{H}\nabla)A_g = 0$, we have

$$\begin{gathered}(\mathbf{V}_e\nabla)A_g = (-(\mathbf{e} \times \nabla\psi_\omega) - \alpha\rho^{-1}\mathbf{j})\nabla A_g\\ \approx (-(\mathbf{e} \times \nabla\psi_\omega) + \alpha(\mathbf{e} \times \nabla H_g))\nabla A_g = -(\mathbf{e} \times \nabla\{\psi_\omega - \alpha P + \alpha(p_e + p_i)\})\nabla A_g.\end{gathered}$$

For the function $\psi_a = \psi_\omega - \alpha P$, it follows from (19) and (20) that

$$\Delta\psi_a = -\omega + \alpha\frac{\partial V_k}{\partial x_l}\frac{\partial V_l}{\partial x_k}. \tag{22}$$

Thus, for A_g we have

$$\frac{\partial A_g}{\partial t} - (\mathbf{e} \times \nabla\psi_a + \alpha\mathbf{e} \times \nabla(p_e + p_i))\nabla A_g = -\nu j_g. \tag{23}$$

We emphasize that the complete pressure-containing term in (23) is by no means associated with p_e in the Ohm's generalized law. It appears owing to the freezing of a magnetic field into the electron plasma component, i.e., owing to the expression $\alpha\rho^{-1}\mathbf{j} \times \mathbf{H}$ in the Ohm's generalized law. The appearance of this term is caused by the fact that the approximation of a large toroidal field allows one to express the poloidal current through the plasma pressure.

For simplicity, we assume that, owing to the large thermal diffusivity along the magnetic field, the temperatures T_e and T_i are constant along the field, i.e., $T_{e,i} = T_{e,i}(A_g, t)$. For tokamaks, T_e along the field

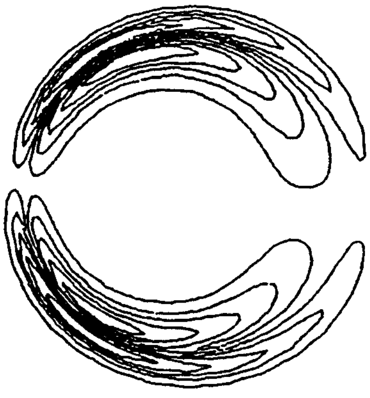

Fig. 1

Fig. 2

is constant with high accuracy. For ions, this is not, generally speaking, the case, although the ion thermal diffusivity along the magnetic field is large. The term $(\mathbf{e}\times\nabla p_{i,e})\nabla A_g = T_{i,e}(\mathbf{e}\times\nabla\rho)\nabla A_g + \rho(\mathbf{e}\times\nabla T_{i,e})\nabla A_g = T_{i,e}(\mathbf{e}\times\nabla\rho)\nabla A_g \sim R^{-4}$ in (23), and it may be ignored. We emphasize that a correct taking into account of $\nabla\rho$ requires an account of $\nabla\rho$ in Eq. (13) as well. This means that, in this case, it is possible to use system (11)–(15), and the equation for A_g turns out to be not simple (see the Appendix).

As a result, we derive the closed system (19)–(23), which does not call for knowledge of a concrete temperature distribution.

Equations (19)–(23) have the following property: if $\rho(t,\mathbf{r})$, $\mathbf{V}(t,\mathbf{r})$, and $A_g(t,\mathbf{r})$ are the solutions of (19)–(23) for $R = R_*$, $\alpha = \alpha_*$, $\nu = \nu_*$, and the initial data (8), the functions $\rho(R_*t,\mathbf{r})$, $R_*\mathbf{V}(R_*t,\mathbf{r})$, and $R_*A_g(R_*t,\mathbf{r})$ will be the solutions of these equations for $R = 1$, $\alpha = \alpha_*/R_*$, and $\nu = \nu_*R_*$, which allows us to confine ourselves to a study of the case $R = 1$.

System (19)–(23) was solved numerically. Computations show that the formula $\rho \approx 1 + \alpha\omega$ holds with high accuracy for not too large α. The quantity ω grows in the process of reconnection, and it reaches a maximum at the moment of complete reconnection and then decreases to zero. Figure 1 illustrates the typical distribution of ω. The picture of the magnetic lines of force closed on themselves is depicted in Fig. 2. The asymmetry in the distribution of ω is associated with the presence of the Hall terms: the asymmetry is the more significant the larger the coefficient α.

On the whole, the reconnection pattern for $\alpha \neq 0$ differs little from the Kadomtsev model, since we use small values of α for the applicability of system (19)–(23). These computations, however, make it possible to get an idea of the density variation depending on ν and α. We shall give estimates for a TEXTOR tokamak. For this facility, we have $\alpha = 0.05$, $R = 3.8$, and $\nu = 10^{-8}$. Computations show that already for $\nu = 2.5\cdot 10^{-6} \gg 10^{-8}$, the maximum difference in time and space of ρ from unity reaches 20%. The value of ω and, hence, the difference of ρ from unity grows with increasing ν. Thus, the role of the effects that we have discussed for the parameters of a real facility can only become more significant.

APPENDIX

We have shown that, for the plasma parameters that are typical of tokamaks, the density variation in developing the tearing mode can be large. The region of the most pronounced variation is concentrated in a close vicinity of the neutral layer. Therefore, the density gradients are large, which can significantly change the flow pattern. A correct allowance for $\nabla\rho$ assumes the use of Eqs. (11)–(15).

We shall show the form of the equation for the vector potential of system (11)–(15). We introduce the vorticity $\omega = (\mathbf{e}\,\mathrm{rot}\,\mathbf{V})$ and the function ψ: $\mathbf{V}_0 = -(\mathbf{e}\times\nabla\psi)$ [see (16)]. For ψ, we have

$$\Delta\psi = -\omega + \alpha\,\mathrm{div}\,\frac{j_g\nabla A_g - \nabla(p_e + H_g)}{\rho}. \tag{24}$$

Using the divergence of (11), we obtain

$$\operatorname{div}\frac{j_g\nabla A_g-\nabla(p_e+H_g)}{\rho}=\operatorname{div}\frac{\nabla p_i}{\rho}+\frac{d}{dt}\operatorname{div}\mathbf{V}+\frac{\partial V_k}{\partial x_l}\frac{\partial V_l}{\partial x_k}. \tag{25}$$

Here $d/dt=\partial/\partial t+(\mathbf{V}\nabla)$. Since $\operatorname{div}\mathbf{V}=-\rho^{-1}(d\rho/dt)$, we have $d(\operatorname{div}\mathbf{V})/dt=-d^2(\ln\rho)/dt^2$. Substituting the latter into the expression $(\mathbf{V}_e\nabla)A_g=(\mathbf{V}+\alpha\rho^{-1}(\mathrm{e}\times\nabla H_g))\nabla A_g$, with allowance for $(\mathbf{H}\nabla)A_g=0$ and (16), (24), and (25), we write

$$\frac{\partial A_g}{\partial t}+(\mathbf{V}_a\nabla)A_g=-\nu j_g,$$

where

$$\mathbf{V}_a=(\mathrm{e}\times\nabla\Delta^{-1}\omega)-\alpha\rho^{-1}(\mathrm{e}\times\nabla p_e)-\alpha\left(\mathrm{e}\times\nabla\Delta^{-1}\left(\operatorname{div}\frac{\nabla p_i}{\rho}\right)\right)-\alpha\left(\mathrm{e}\times\nabla\Delta^{-1}\left(\frac{\partial V_k}{\partial x_l}\frac{\partial V_l}{\partial x_k}-\frac{d^2(\ln\rho)}{dt^2}\right)\right)$$

and Δ^{-1} is the Laplace inverse operator. Clearly, taking into account the density gradient is not simple.

We note that the differences of a real toroidal geometry from the spiral one that we have used are manifested also in the second-order expansion in the parameter R. Factors such as electron inertia [10], longitudinal electron viscosity [12], etc. can affect the character of sawtooth oscillations. However, the conclusion that a significant density variation in the development of the kink-tearing instability in tokamaks' plasma is possible is valid in these cases as well. This conclusion follows from the fact that $\operatorname{div}\mathbf{V}$ turns out to be of the order of $\alpha\mathbf{H}\nabla j_z$. In the case of high conductivity, the value of j_z is large and, therefore, the plasma cannot be considered incompressible. The expression for $\operatorname{div}\mathbf{V}$ can be derived from the equation for H_z (this is not obligatorily the case of a helical geometry) under the assumption of the smallness of $\partial H_z/\partial t$ compared with the other terms of this equation. This is true if the characteristic time of the process is much longer than the Alfvén time a/V_A, which corresponds to reality. The assumption $R^{-1}\ll 1$ is not obligatory.

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